Self-managing learning of vector in $R^2$ with support of GeoGebra

Aprendizaje autogestivo de vectores en $R^2$ con apoyo de GeoGebra

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Abstract
In this work, the research problem is related to the design, implementation and effectiveness of an Object for self-managed Learning on the learning results of vectors in $R^2$. The research was of a quasi-experimental type, due to the fact that the group with which we worked was already formed previously, from which two subgroups were randomly selected, one called control and the other experimental, its approach was quantitative when comparing the learning results through a post-test and a satisfaction survey. The objective of the research was to determine and evaluate the effect produced with the use of the Learning Object generated in GeoGebra on the learning results obtained by the students. According to the result in the statistical analysis, it was concluded that there was no statistically significant difference between the learning results of the students of the experimental group and those of the control group.

Keywords: Self-managed learning, GeoGebra, learning object, vectors in $R^2$.

Resumen
En este trabajo el problema de investigación se relaciona con el diseño, implementación y efectividad de un Objeto para Aprendizaje autogestivo sobre los resultados de aprendizaje de vectores en $R^2$. La investigación fue de tipo cuasi-experimental, debido a que el grupo con el que se trabajó ya estaba formado previamente, del mismo se seleccionaron dos subgrupos aleatoriamente, uno denominado control y el otro experimental, su enfoque fue cuantitativo al comparar los resultados de aprendizaje por medio de un postest y de una encuesta de satisfacción. El objetivo de la investigación fue el de determinar y evaluar el efecto producido con el empleo del Objeto para Aprendizaje generado en GeoGebra sobre los resultados de aprendizaje obtenidos por los alumnos. Acorde al resultado en el análisis estadístico, se concluyó que no existió diferencia estadísticamente significativa entre los resultados de aprendizaje de los alumnos del grupo experimental y los del grupo control.

Palabras clave: Aprendizaje autogestivo, GeoGebra, objeto para aprendizaje, vectores en $R^2$. 
Introduction

Problematic situation

The main objective of the school should be to provide various tools, in addition to directing students in the necessary preparation so that they can adequately fulfill the social objectives of the institution. In particular, with the teaching of mathematics in any educational institution, it is about students being able to function in society, which contains, among other things, the ability to reason logically, as well as being able to pose problems that are not routine and manifest for those ideas that are based on mathematics.

However, in order to be able to deal in a flexible and versatile way with the different conditions in the school that a teacher faces in his academic life, Crispín, et al. (2012, p. 6) notes that "teachers adopt new teaching strategies that promote meaningful and active learning, where students feel capable of learning, working as a team, applying what they experience in different contexts, knowing how to make decisions and act accordingly”.

In this sense Cabero (2000) affirms that:

The figure of the teacher is fundamental in the success of the incorporation of Information and Communication Technologies (ICT). Your thoughts and the activities with which you take advantage of technologies, your disposition, your experiences of success or failure towards its integration and the possible benefits in the teaching and learning processes are the central axis on which a good practice pivots with the ICT. (p. 5).

According to Fernández and Lazaro (2008, p. 180) is required for a quality education based on the unification of ICT, in addition to “teacher training that makes possible a shift in the educational approach, in their ways of relating and interacting with them and, a substantial alteration in the modes, thoughts, representations and practices developed around the technologies”.

It is then that, with the support of ICT, a Learning Object (LO) was developed in GeoGebra (Hohenwarter, 2015) of the characteristics of vectors in $R^2$ particularly magnitude, direction and projection. In addition, the program ExamView Test Generator® (Broderick, 2013)
was used for the elaboration of both the self-assessment and the post-test for the characteristics of vectors.

To make the LO, both pedagogical factors and software aspects were taken into account to obtain quality educational material in order to achieve the learning objectives. While in the construction of the same one began from the analysis of the learning needs. The results were used to delimit the LO, select and structure the content, and design the activities to be carried out.

**Antecedents**

**Self-managed learning**

According to Cajal (2020):

The self management of learning, also called self-managed learning, self-regulated learning or self-managed, is the active and constructive process by which students establish and work toward learning goals through the monitoring, regulation, and control of motivation, cognition and behavior. (p. 1).

In addition, Cajal (2020, p. 2) mentions that in self-managed learning “it is understood that the student himself manages all these aspects of himself to achieve the objetives that he has set for himself and, in addition, the objectives are also fed back with personal aspects of the student”.

Smith (2001, p. 665) affirms that “as opposed of the students who remain passive receiving the instruction that is determined by an external authority, the student who self-manages his motivation has an intrinsic or extrinsic objetive that guides him towards self-improvement”. Ie, according to Góngora (s. f.) “both the student and the teacher demand from themselves and from each other greater personal motivational resources to manage and perform in academic activities” (par. 11).

**Learning Objects**
The LO are in the educational environment are implemented without taking ICT into account on many occasions, however, when they are used, the concept of having some self-contained learning unit makes sense, interoperable, reusable, durable and upgradeable.

There are two areas of knowledge that are related to the LO, pedagogical and technological. The first has to do with the pedagogical questions associated with designing and developing educational content based on the notion of LO, which have to be functional in various learning environments; the second comprises the challenges related to technology during the creation of educational platforms and programs based on this notion.

Therefore, according to Mora (2012, p. 108), an LO is:

An extensive entity, but in reality, it must develop only one objective, it must contain activities and a final evaluation or verification. This can be varied according to specific needs. It is a specific material, with contents and evaluation activities. This is for the purpose to expand the possibilities of adapting the object for various courses and contexts, as opposed to a material that is more extensive and covers many contents.

Objective

The objective of this research was to determine and evaluate the effect produced by the use of the LO on the learning results obtained by the students in the course of Linear Algebra I, on the characteristics magnitude, direction and projection of vectors in $\mathbb{R}^2$.

In addition, the research problem is related to the design, implementation and effectiveness of the LO in the learning results of the characteristics magnitude, direction and projection of vectors in $\mathbb{R}^2$ in the students of the course of Linear Algebra I who study in the different careers of the University Center of Exact Sciences and Engineering (CUCEI) from the University of Guadalajara (UdeG), since according to observations commented by the professors of the course in the academy meetings, such a subject is complicated for the students.

In addition, this research was aimed at answering the research question: ¿what effect does the implementation of the LO of the characteristics magnitude, direction and projection of vectors in $\mathbb{R}^2$ on the learning outcomes of the students in the course of Linear Algebra I taught at the CUCEI?
Materials and method

Participants

The research was carried out in a group of the course of Linear Algebra I, offered by the Department of Mathematics (DM) of the CUCEI of the UdeG, which is studied in different semesters and different careers, due to the fact that there are no prerequisites, students enroll with the professor and schedule that best suits them. As the group was already formed beforehand, the research was quasi-experimental.

In this group, two subgroups were randomly selected, one was a control group and another experimental group, for the latter a self-managed LO designed in GeoGebra was developed (Hohenwarter, 2015). The experimental group consisted of 12 students and the other control group, while the experimental group consisted of 13 students.

Students in the experimental group were provided with a CD with the LO designed for this research, and students in the control group attended classes in the traditional treatment.

According to the study plan, the course of Linear Algebra I is considered as a course-workshop, which is assigned 60 hours of workload per semester, of which 40 hours are theory and 20 practice, which are taught in two classes per week, of two hours each respectively.

Technique and instruments

The research was carried out in three stages:

For the first stage of this work, an intensive bibliographical analysis was made, in order to document the research to be carried out, as well as the delimiting of the object of study, essential in the realization of the experimental design.

The second stage was destined to design and elaborate the necessary materials for the implementation of the proposal of the LO for the students assigned to the experimental group, in addition to the satisfaction survey on the quality of the material that was applied to the students of the experimental group and the post-test for the control and experimental groups.

Finally, in the last stage, the information was processed to analyze it and obtain the results and conclusions.
Learning Object

The material designed for the LO was recorded on a CD, since there was no Internet domain for its hosting. It was designed using GeoGebra applets (Hohenwarter, 2015) and displayed through a web page. In addition, the program ExamView Test Generator® (Broderick, 2013) was used for the elaboration of both the self-assessment and the post-test for the characteristics magnitude, direction and projection of vectors in vectors in $R^2$.

The LO contained several sections (Figure 1), the first called Vectores, is an introduction to the subject of vectors in $R^2$ (Figure 2).

Figure 1.

Screenshot of the LO Sections.

![Screenshot of the LO Sections](image)

Source: own elaboration.

Figure 2.

Screenshot of part of the content of the Vectores section.

![Screenshot of part of the content of the Vectores section](image)

Source: own elaboration.
The next three correspond to the characteristics magnitude, direction and projection of vectors in $R^2$. The examples that were presented for each one of the characteristics can be verified with the corresponding applet.

The Figure 3 shows a screenshot with part of the content of the Magnitud section and the GeoGebra applet (Hohenwarter, 2015).

**Figura 3.**
*Screenshot of part of the content of the Magnitud section and the GeoGebra applet.*

![Screenshot of part of the content of the Magnitud section and the GeoGebra applet.](image)

*Source: own elaboration.*

The Figure 4 shows a screenshot with part of the content of the Dirección section and the GeoGebra applet (Hohenwarter, 2015).
Figure 4.
Screenshot of part of the content of the Dirección section and the GeoGebra applet.

Source: own elaboration.

The Figure 5 shows a screenshot with part of the contents of the Proyección section and the GeoGebra applet (Hohenwarter, 2015).

Figure 5.
Screenshot of part of the content of the Proyección section and the GeoGebra applet.

Source: own elaboration.

The Figure 6 shows a screenshot with part of the Aplicación content, in which some examples are presented in which the characteristics of vectors can be applied in $R^2$. The Figure 7 shows a screenshot with part of the introduction to the Actividades section, which contains
various problems and exercises, in addition to the GeoGebra applet (Hohenwarter, 2015) with which you can obtain or verify the results.

**Figure 6.**
*Screenshot of part of the content of the Aplicación section.*

![Screenshot of part of the content of the Aplicación section.](source)

*Source: own elaboration.*

**Figure 7.**
*Screenshot of part of the content of the Actividades section and the GeoGebra applet.*

![Screenshot of part of the content of the Actividades section and the GeoGebra applet.](source)

*Source: own elaboration.*

Finally, in the Autoevaluación section, a series of questions were presented, which were used so that students, independently, self-assess what they have learned about the characteristics
of vectors magnitude, direction and projection in $\mathbb{R}^2$. The Figure 8 shows part of the content of this section.

**Figure 8.**

*Screenshot of part of the content of the Autoevaluación section.*

![Screenshot of part of the content of the Autoevaluación section](image)

*Source:* own elaboration.

For the realization of this section was used the program ExamView Test Generator® (Broderick, 2013).

**Satisfaction survey**

The satisfaction survey was applied to the experimental group (see appendix 1) after answering the post-test. The values of each criterion were made using the Likert scale (Table 1).

**Table 1.**

*Values and criteria for each question in the satisfaction survey.*

<table>
<thead>
<tr>
<th>Value</th>
<th>Criteria</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Definitely yes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>More or less</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>No</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Definitely not</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Source:* own elaboration.

This contained 14 questions to evaluate, which focused on knowing the satisfaction of the students regarding the use of the LO designed in GeoGebra (Table 2).
Table 2.

Questions to evaluate in the satisfaction survey.

<table>
<thead>
<tr>
<th>Num</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>¿The LO covered the topic adequately?</td>
</tr>
<tr>
<td>2</td>
<td>¿The theory in the LO was sufficient for the topic?</td>
</tr>
<tr>
<td>3</td>
<td>¿The theory in the LO was understandable?</td>
</tr>
<tr>
<td>4</td>
<td>¿The examples presented were understandable?</td>
</tr>
<tr>
<td>5</td>
<td>¿The examples presented in the LO were sufficient?</td>
</tr>
<tr>
<td>6</td>
<td>¿Do you think the examples were useful to learn the topic?</td>
</tr>
<tr>
<td>7</td>
<td>¿The time that has to be dedicated to the LO is sufficient?</td>
</tr>
<tr>
<td>8</td>
<td>¿The self-assessment exercises were understandable?</td>
</tr>
<tr>
<td>9</td>
<td>¿In relation to the number of hours dedicated to the LO, the material is excessive?</td>
</tr>
<tr>
<td>10</td>
<td>¿Working with the LO left you satisfied?</td>
</tr>
<tr>
<td>11</td>
<td>¿How do you rate the LO design?</td>
</tr>
<tr>
<td>12</td>
<td>¿Would you recommend your classmates to use this LO?</td>
</tr>
<tr>
<td>13</td>
<td>¿The self-assessment helped you in learning the topics?</td>
</tr>
<tr>
<td>14</td>
<td>¿The quality of the material is good?</td>
</tr>
</tbody>
</table>

Source: own elaboration.

Finally, a section with three open questions for comments and suggestions from students was included.

- Mentions that the LO is missing.
- Based on what makes up the LO, ¿what helped you the most to learn the topics covered?
- Write a comment or suggestion

Post-test

The purpose of the post-test was to evaluate the learning results of the students in the characteristics magnitude, direction and projection of vectors in $R^2$. It consisted of 10 exercises divided into three sections. The first was to answer true or false and contained three exercises, the second consisted of four multiple choice exercises and the third of three problems to find the correct answer. The Figure 9 shows the post-test applied to both the experimental and control groups.

Figure 9.

Post-test of the characteristics magnitude, direction and projection of vectors in $R^2$. 
POSTEST

Nombre: ____________________________

Apellido paterno  Apellido materno  Nombres

Anota en la línea de la izquierda una F si el enunciado es falso o una V si es verdadero.

   ____ 1. La magnitud es todo lo que se puede medir
   ____ 2. Los vectores \( \vec{A} \) y \( \vec{C} \) tienen el mismo módulo, siendo \( \vec{A} = (2, 1) \), \( \vec{B} = (4, 2) \), \( \vec{C} = (0, -4) \) y \( \vec{D} = (-1, -2) \)
   ____ 3. Los elementos de un vector, son la magnitud, dirección y sentido

Selecciona la opción que responde correctamente a cada pregunta o enunciado y escribe la letra correspondiente en la línea de la izquierda.

   ____ 4. La dirección del vector \((4, 8)\), es:
   A) \( \pi \)  B) \( \tan^{-1}(8 - 4) \)  C) \( \frac{8}{4} \pi \)  D) \( \tan^{-1}\left(\frac{8}{4}\right) \)

   ____ 5. Hallar el extremo del vector sabiendo que su origen es el punto
   A) \((-3, 0)\)  B) \((2, 3)\)  C) \((0, 3)\)  D) \((-2, 1)\)

   ____ 6. La proy \( \overrightarrow{w} \) es igual a:
   A) \( \frac{u \cdot w}{|w|} \)  B) \( \frac{w}{|w|} \)  C) \( \frac{u \cdot w}{|w|} \)  D) \( \frac{u \cdot w}{|u|} \) \( \overrightarrow{u} \)

   ____ 7. Teniendo el vector \( \vec{u} = 3\vec{a} - 2\vec{b} \), dónde \( \vec{a} = (-1, 2) \) y \( \vec{b} = (5, -2) \). Obtén las coordenadas de \( \vec{u} \).
   A) \((-10, -7)\)  B) \((-7, 2)\)  C) \((2, 13)\)  D) \((-13, 10)\)

Resuelve los siguientes problemas

8. Calcula el valor de \( k \) sabiendo que la magnitud del vector \( \vec{v} = (k, 3) \) es 5

9. Sean \( P = (2, 3) \), \( Q = (5, 7) \), \( R = (2, -3) \) y \( S = (1, 2) \). Calcula la proy \( \overrightarrow{F \overrightarrow{Q} \overrightarrow{R}} \)

10. Un avión vuela hacia el norte a 90m/s, un fuerte viento sopla hacia el este a razón de 20 m/s y desvía su rumbo. Hallar la velocidad del avión para un observador en la tierra.

Source: own elaboration.

Process
The experimental phase was carried out for 9 hours and 10 minutes, which were divided into five sessions, each lasting one hour and 50 minutes. Table 3 shows the activities carried out in each of the sessions.

**Tabla 3.**

*Activities carried out in each session.*

<table>
<thead>
<tr>
<th>Num. session</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>In the first hour of this session, the students of the experimental group were taken to the computer center where each one was given a CD, which contained the LO designed in GeoGebra and an explanation of its content was given, how to use it and navigate in it. They were given the topics that they should study in a self-managed way. The rest of the time, students were asked to navigate the LO to familiarize themselves with its content.</td>
</tr>
<tr>
<td>2</td>
<td>This session focused on students self-managed, studying the sections on Vectors, Magnitude, Direction and Projection. The last 20 minutes were used to resolve some questions that the students had.</td>
</tr>
<tr>
<td>3</td>
<td>This session was intended for the students, again in a self-managed way, to study the Aplicación section of the magnitude, direction and projection characteristics of vectors in $R^2$. The last 20 minutes were also used to answer the questions of the students.</td>
</tr>
<tr>
<td>4</td>
<td>During the first hour of this session, students were instructed to continue self-studying the Actividades section and the remaining time they were asked to enter and answer the content in the Autoevaluación section, so that they could autonomously self-assess what they had learned.</td>
</tr>
<tr>
<td>5</td>
<td>The first hour and a half was devoted to answering the post-test. The remaining time was used for the students to answer the satisfaction survey.</td>
</tr>
</tbody>
</table>

*Source: own elaboration.*

**Results and discussions**

**Analysis of achievement by students**

To obtain the results on the achievement of the students of the control and experimental groups in the characteristics magnitude, direction and projection of vectors in $R^2$, the data obtained from the post-test evaluation were analyzed. Table 4 shows the results of the average obtained by both the students of the experimental group and the control group.

**Table 4.**
Post-test scores of students in the experimental and control group.

<table>
<thead>
<tr>
<th>Student</th>
<th>Experimental group</th>
<th>Control group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>Average</td>
</tr>
<tr>
<td>1</td>
<td>50</td>
<td>75</td>
</tr>
<tr>
<td>2</td>
<td>55</td>
<td>75</td>
</tr>
<tr>
<td>3</td>
<td>55</td>
<td>70</td>
</tr>
<tr>
<td>4</td>
<td>65</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>65</td>
<td>65</td>
</tr>
<tr>
<td>6</td>
<td>45</td>
<td>55</td>
</tr>
<tr>
<td>7</td>
<td>65</td>
<td>75</td>
</tr>
<tr>
<td>8</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>9</td>
<td>40</td>
<td>65</td>
</tr>
<tr>
<td>10</td>
<td>65</td>
<td>30</td>
</tr>
<tr>
<td>11</td>
<td>70</td>
<td>60</td>
</tr>
<tr>
<td>12</td>
<td>45</td>
<td>55</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>55</td>
</tr>
</tbody>
</table>

Average 55.42  58.85

Source: own elaboration.

Statistical results of the post-test

The results of the post-test show that the control group achieved a better performance than the experimental group, but this happened in the sample taken, to confirm the equivalence of two variances, the test F of Fisher Snedecor was performed. The F statistic is equal to the variance of the first sample divided by the variance of the second sample, that is:

\[ F = \frac{S^2_1}{S^2_2} \]

where: \( S^2_1 \) is the variance of the experimental group and \( S^2_2 \) is the variance of the control group. In addition, we have that the statistic uses a distribution \( F \) with \( n_1-1 \) y \( n_2-1 \) degrees of freedom.

With a significance level \( \alpha = 0.05 \), the null hypothesis establishes the equivalence of variances of two populations \( H_0: \sigma^2_1 = \sigma^2_2 \) against the alternative hypothesis \( H_1: \sigma^2_1 \neq \sigma^2_2 \).
For the results obtained by the students of the experimental and control group in the post-test, it was done with the program Stathgraphics® (Polhemus, 2014). The F test for variances of two samples is observed in Table 5.

**Table 5.**

*Two-sample analysis of variance.*

<table>
<thead>
<tr>
<th>Source of variations</th>
<th>Sum of squares</th>
<th>Degrees of freedom</th>
<th>Square average</th>
<th>ration-F</th>
<th>value-P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>44.1603</td>
<td>1</td>
<td>44.1603</td>
<td>0.286122</td>
<td>0.5979</td>
</tr>
<tr>
<td>Intra groups</td>
<td>3549.84</td>
<td>23</td>
<td>154.341</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>3594.0</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Source: own elaboration.*

Table 5, also known as ANOVA, decomposes the variance of qualifications into two parts: one called between groups and another intra groups. The ration-F obtained is equal to 0.286122, being the result of dividing the between-group estimate by the intra-group estimate. Since the value-P of the ration-F is equal to 0.5979 being greater than or equal to 0.05, then it is considered that there is no statistically significant difference between the mean of the qualifications obtained between one group level and another, that is to say that the achievement of the group with or without the use of the LO did not show a variation, with a confidence level of 95.0%.

The software Statgraphics® (Polhemus, 2014) it was used to verify the research hypothesis, in particular the comparison of means. For this purpose, the test statistic *t*-student was used with a level of significance of $\alpha = 0.05$. In the comparison of means, a value of $t = -0.686604979248934$, to which corresponds the probability value of $P = 0.24959806$.

Comparing it with the value $\alpha$, it is obtained that $P > \alpha$, therefore it is not possible to reject the null hypothesis regarding the equality of the means of both groups.

Due to the above, the null hypothesis cannot be rejected, due to this it is considered that there is no statistically significant difference between the means of the samples. Consequently, the null hypothesis is accepted, that is, with the use of the LO designed in GeoGebra (Hohenwarter, 2015) the learning outcomes of students in the experimental group are less than or equal to the learning outcomes of students in the control group.
Analysis of the answers to the questions of the opinion survey

Table 6 shows the values obtained in each of the answers corresponding to the questions of the satisfaction survey designed for the students of the experimental group, this according to the established scale.

**Tab6 6.**

*Values for the answers corresponding to each question.*

<table>
<thead>
<tr>
<th>Student</th>
<th>Question</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
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</table>

*Source: own elaboration.*

While the percentage corresponding to each question of the satisfaction survey elaborated for the experimental group according to the determined scale is shown in the Table 7.

**Table 7.**

*Percentage corresponding to each question.*

<table>
<thead>
<tr>
<th>Question</th>
<th>Definitely yes</th>
<th>Yes</th>
<th>More or less</th>
<th>No</th>
<th>Definitely not</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25.00</td>
<td>75.00</td>
<td>0.00</td>
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Journal EDUCATECONCIENCIA. Vol. 30, Num. 34 quarterly publication january- march 2022
https://doi.org/10.58299/edu.v30i34.501
According to the answers obtained by the students, it can be seen that the total consider that the material designed for the LO was sufficient and covered the subject in an adequate way, in addition to the theory contained being understandable as well as the examples presented in the corresponding section.

With respect to the point about the number of examples presented, the opinion is divided, since, although the majority of the students consider that they were enough, others think that a few more were needed. The same happens with their usefulness, since the opinion is divided between very useful and useful.

With respect to the time that has to be dedicated to the LO, it was possible to perceive that, although the majority of the students consider that it is sufficient, two students consider that a little more is needed. Likewise, according to the established time, the material contained in the LO was not excessive.

In relation to the self-assessment, all the students consider that the exercises presented in this section were understandable and that they helped them learn the subject.

In addition, the students consider that the quality of the content of the LO goes from good to excellent, but that it needs to improve a little in its design, although working with it left them satisfied and that they would recommend it.
Finally, the main comments of the students regarding each of the three open questions of the satisfaction questionnaire about their work with the LO are presented.

- Mentions that the LO is missing.
  The comments of the students to this question were varied, there were two who reserved their opinion, while other students think that the material was good and understandable and that, if more topics could be included, some others think that the size of the applet would be larger and some others that a GeoGebra manual would be nice for future handling of this.

- Based on what makes up the LO, ¿what helped you the most to learn the topics covered?
  Three students think that the self-assessment helped them a lot, another four think that the didactic activities, one thinks that the use of GeoGebra and the topics separately, another thinks that the links since it seems to him a more didactic activity, another comments that the explanation with images and finally they comment that the exercises in general and the type of explanation.

- Write a comment or suggestion
  For this question, there were three students who reserved their opinion, while other students only mentioned that they thought it was good, that it helped them a lot, that it is very good material and others that the interface could be improved a little more.

**Answer to the research question**

Next, the research question will be answered: ¿what effect does the implementation of the LO of the magnitude, direction and projection characteristics of vectors in \( R^2 \) on the learning outcomes of the students in the course of Linear Algebra I taught at the CUCEI?, this based on the results of the statistical analysis that was carried out as a consequence of the application of the LO on the subject of the characteristics magnitude, direction and projection of vectors in \( R^2 \).

When taking into account the statistical results of the post-test, it was found that there is no positive effect on the learning of the experimental group, from the application of the LO of the characteristics magnitude, direction and projection of vectors in \( R^2 \) with the support of GeoGebra (Hohenwarter, 2015) which is confirmed by the result obtained through a hypothesis test in which the means of the experimental and control groups were compared.
It was found that there is no statistically significant difference between the means of these groups when the corresponding analysis was carried out with Statgraphics® (Polhemus, 2014), in which the statistic test $t$-student was used for the comparison of the means, obtaining a probabilistic value $P = 0.24959806$ and a value of $t = -0.686604979248934$ with a significance level $\alpha = 0.05$. Therefore, the learning results of the experimental group were not better in the subject of characteristics magnitude, direction and projection of vectors in $R^2$ compared to the control group.

**Conclusions**

According to the answers of the exam that was applied to the students, both from the experimental group and the control group, the use of the LO designed in GeoGebra and the corresponding statistical analysis, there was no statistically significant difference between the learning results of the students who worked with the material in a self-managed way and those who attended classes in a traditional treatment.

And according to the previous point, it is worth mentioning that many of the students do not have the habit of studying autonomously, so it is very easy to get distracted by the slightest event. In addition, for this research they were not asked to participate voluntarily in its experimentation, so it can be inferred that when they do not study self-managed on their own and voluntarily, as in this case, the learning results will not be the desired ones.

Even when the use of the LO material did not produce better desired learning results in the students who used it, can be considered that the material contained in the LO will be useful when students want to learn on their own, since most consider that its quality was good to excellent and that its content is understandable, in addition to recommending it for future courses.

It is important to keep in mind that for the preparation of material containing an LO, learning theories must be known and the contents structured so that the theory, examples, exercises, self-assessment and post-test, without neglecting what type of material is the most appropriate so that it is as clear and understandable as possible, and thus achieve the main objective of student learning.

**Referencias**


Appendix 1

SATISFACTION SURVEY ON THE LO

Instructions: This satisfaction survey is about your experience using the LO. Mark with an X the value that you consider the most appropriate in each criterion according to the indicated scale. Be HONEST.

Scale

<table>
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<tbody>
<tr>
<td></td>
<td>Definitely yes</td>
<td>Yes</td>
<td>More or less</td>
<td>No</td>
<td>Definitely not</td>
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</table>

<table>
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<tr>
<th>Opinions</th>
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<td>1. ¿The LO covered the topic adequately?</td>
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<td>2. ¿The theory in the LO was sufficient for the topic?</td>
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<td>3. ¿The theory in the LO was understandable?</td>
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<td>4. ¿The examples presented were understandable?</td>
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<td>5. ¿The examples presented in the LO were sufficient?</td>
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<td>6. ¿Do you think the examples were useful to learn the topic?</td>
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<td>7. ¿The time that has to be dedicated to the LO is sufficient?</td>
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<td>8. ¿The self-assessment exercises were understandable?</td>
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<td>9. ¿In relation to the number of hours dedicated to the LO, the material is excessive?</td>
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<td>10. ¿Working with the LO left you satisfied?</td>
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<td>11. ¿How do you rate the LO design?</td>
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<td>12. ¿Would you recommend your classmates to use this LO?</td>
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<td>13. ¿The self-assessment helped you in learning the topics?</td>
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<td>14. ¿The quality of the material is good?</td>
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Mentions that the LO is missing

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Based on what makes up the LO, ¿what helped you the most to learn the topics covered?

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Write a comment or suggestion

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